HQT: Notation, Glossary and Mathematical Definitions

In the following some material for reference (notation, glossary, and mathematical definitions) as well as for review (mathematical definitions) is given.

This material is constantly updated (August 4, 2005).

The notation in ***Appendix 1*** describes the symbols used in our publications concerning *Heim Quantum Theory* (*HQT*).

The glossary of ***Appendix 2*** describes the special terminology used in *HQT*.

The glossary of ***Appendix 3*** describes and explains the special mathematical terminology used in *Heim's* original work.

The mathematical definitions in ***Appendix 4*** refer to definitions used in modern physics, and are meant to facilitate the reading of our papers.

Appendix 1: Notation and Physical Constants

$\tilde{A}$ value for the onset of conversion of photons into gravitophotons.

A denotes the strength of the shielding potential caused by virtual electrons.

Compton wave length of the electron

$$\lambda_C = \frac{h}{m_e c} = 2.43\times 10^{-12} m\,, \quad \lambda\!\!\!^{-}_C = \lambda_C / 2\pi\,.$$

c speed of light in vacuum 299,792,458 m/s ,

$(1/c^2 = \varepsilon_0\, \mu_0)$.

D diameter of the primeval universe, some 10^{125} m that contains our optical universe.

D_O diameter of our optical universe, some 10^{26} m.

d diameter of the rotating torus.

d_T vertical distance between magnetic coil and rotating torus.

-e electron charge -1.602×10^{-19} C.

$\hat{e}_z$ ***unit vector*** in z-direction.

F_e electrostatic force between 2 electrons.

F_g gravitational force between 2 electrons.

F_{gp} gravitophoton force, also termed Heim-Lorentz force, $\boldsymbol{F}_{gp} = \Lambda_p e \mu_0 \boldsymbol{v}^T \times \boldsymbol{H}$.

$\boldsymbol{G = G_g + G_{gp} + G_q}$ $= 6.67259 \times 10^{-11}$ m^3 kg^{-1} s^{-2},

gravitational constant [1].

G_g graviton constant, $G_g \approx G$ that is G_g decribes the gravitational interaction without the postulated gravitophoton and quintessence interactions.

G_{gp} gravitophoton constant, $G_{gp} \approx (1/67)^2 G_g$.

G_q quintessence constant, $G_q \approx 4 \times 10^{-18} G_g$.

$g_{ik}^{(gp)}$ ***metric subtensor for the gravitophoton*** in subspace $I^2 \cup S^2$ (see *glossary* for subspace description).

$g_{ik}^{(ph)}$ ***metric subtensor for the photon*** in subspace $I^2 \cup S^2 \cup T^1$ (see *glossary* for subspace description).

h Planck constant 6.626076 × 10^{-34} J·s, $\hbar = h/2\pi$.

h_{ik} metric components for an almost flat spacetime.

$\ell_p = \sqrt{\frac{G\hbar^3}{c^3}} = 1.6 \times 10^{-35} m$ ***Planck length***.

m_e electron mass 9.109390 × 10^{-31} kg.

m_0 mass of proton or neutron 1.672623 × 10^{-27} kg and 1.674929 × 10^{-27} kg.

N_n number of protons or neutrons in the universe.

q electric charge.

R distance from center of coil to location of virtual electron in torus.

r_N distance from nucleus to virtual electron in torus.

R_- is a lower bound for gravitational structures, comparable to the Schwarzschild radius. The distance at which gravitation changes sign, ρ, is some 46 Mparsec.

R_+ denotes an upper bound for gravitation and is some type of Hubble radius, but is not the radius of the universe, instead it is the radius of the optically observable universe. Gravitation is zero beyond the two bounds, that is, particles smaller than R_- cannot generate gravitational interactions.

r_e classical electron radius

$$r_e = \frac{1}{4\pi\epsilon_0}\frac{e^2}{m_e c^2} = 3 \times 10^{-15} m.$$

r_{ge} ratio of gravitational and electrostatic forces between two electrons.

v velocity vector of charges flowing in the magnetic coil, some 10^3 m/s in circumferential direction.

v^T bulk velocity vector for rigid rotating ring (torus) (see Sections. 3 and 4), some 10^3 m/s in circumferential direction.

w_{gp} probability amplitude (the square is the coupling coefficient) for the gravitophoton force (fifth fundamental interaction)

$w_{gp}^2 = G_{gp}\frac{m_e^2}{\hbar c} = 3.87 \times 10^{-49}$ probability amplitudes (or coupling amplitudes) can be distance dependent.

w_{gpe} **probability amplitude for emitting a gravitophoton by an electron**

$$w_{gpe} = w_{gp}.$$

w_{gpa} **probability amplitude for absorption of a gravitophoton by a proton or neutron**

$$w_{gpa}^2 = G_{gp} m_p \frac{m_e}{\hbar c}.$$

w_{g_q} conversion amplitude for the transformation of gravitophotons and gravitons into the quintessence particle, corresponding to the dark energy (rest mass of some 10^{-33} eV).

$\boldsymbol{w_{ph}}$ probability amplitude (the square is the coupling coefficient for the electromagnetic force, that is the fine structure constant α)

$$w_{ph}^2 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c} = \frac{1}{137}.$$

$\boldsymbol{w_{ph\_qp}}$ ***conversion amplitude*** for the transformation of photons into gravitophotons.

$\boldsymbol{w_q}$ ***probability amplitude*** for the quintessence particle,(*sixth fundamental interaction*), corresponding to dark energy (rest mass of some 10^{-33} eV).

Z charge number (number of protons in a nucleus of an atom).

$\boldsymbol{Z_0}$ ***impedance of free space***,

$$Z_0 = \sqrt{\frac{\epsilon_0}{\mu_0}} \approx 376.7\,\Omega.$$

α coupling constant for the electromagnetic force or fine structure constant 1/137.

$\boldsymbol{\alpha_{gp}}$ ***coupling constant for the gravitophoton force***.

γ ***ratio of probabilities for the electromagnetic and the gravitophoton force***

$$\gamma = \left(\frac{w_{ph}}{w_{gp}}\right)^2 = 1.87\times 10^{46}.$$

μ_0 ***permeability of vacuum*** $4\pi \times 10^{-7}$ N/m^2 .

τ ***metron* area** (minimal surface $3Gh/8c^3$), current value is 6.15×10^{-70} m^2.

Φ ***gravitational potential***, Φ=GM/R.

ω ***rotation vector***.

Abbreviations

BPP breakthrough propulsion physics

GP Geomtrization Principle

GR General Relativity

GRP General Relativity Principle

HQT Heim Quantum Theory

LQT Loop Quantum Theory

LHS left hand side

ls light second

ly light year

QED Quantum Electro-Dynamics

RHS right hand side

SR Special Relativity

VSL Varying Speed of Light

Subscripts

e electron

gp gravitophoton

gq from gravitons and gravitophotons into quintessence

ph denoting the photon or electrodynamics

sp space

Superscripts

em electromagnetic

gp gravitophoton

ph photon

T indicates the rotating ring (torus)

Note: Since the discussion is on engineering problems, SI units (Volt, Ampere, Tesla or Weber/m^2) are used. 1 T = 1 Wb/m^2 = 10^4 G = 10^4 Oe, where Gauss (applied to ***B***, the magnetic induction vector) and Oersted (applied to ***H***, magnetic field strength or magnetic intensity vector) are identical. Gauss and Oersted are used in the *Gaussian* system of units. In the MKS system, ***B*** is measured in Tesla, and ***H*** is measured in A/m (1A/m = $4\pi \times 10^{-3}$ G).

Note: For a conversion from CGS to SI units, the electric charge and magnetic field are replaced as follows:

$e \rightarrow e/\sqrt{4\pi\epsilon_0}$ and $\boldsymbol{H} \rightarrow \sqrt{4\pi\mu_0}\boldsymbol{H}$.

Appendix 2: Glossary of Physical Terms

aeon Denoting an indefinitely long period of time. The aeonic dimension can be interpreted as steering structure governed by the *entelechial* dimension toward a dynamically stable state.

apeiron the unlimited primeval substance in Greek natural philosophy Used to characterize the state of existence before the quantized bang, similar to Penrose's mathematical world or world of potentialities.

anti-hermetry Coordinates are called anti-hermetric if they do not deviate from Cartesian coordinates, i.e., in a space with anti-hermetric coordinates no physical events can take place.

canonical Conforming to a general rule or acceptable procedure, a canonical form is the simplest form possible (for instance a unit matrix).

condensation For matter to exist, as we are used to conceive it, a distortion from Euclidean metric or condensation, a term introduced by Heim, is a necessary but not a sufficient condition.

condensor The Christoffel symbols of the second kind $\Gamma^{i}_{k\,m}$ become the so called condensor functions, φ^{i}_{km} that are normalizable. It can be shown that φ^{i}_{km} the have tensor character. It should be *noted* that in the eigenvalue equations for the mass spectrum of elementary particles, the φ^{i}_{km} are eigenfunctions and thus must not be confused with the Christoffel symbols. It should be mentioned that Heim first writes a symbolic eigenvalue equation that he later on using symmetry arguments transforms into a mathematically correct eigenvalue equation. The term condensor is derived from the fact that these functions represent *condensations* of the spacetime metric. The condenser is an operator projecting a deformation in the 6, 8, or 12-dimensional metronic lattice into $\mathbb{R}^4$, where it appears as an intricate, geometrically structured, compressed or “condensed” lattice configuration. This condensed, structured region is what eventually is interpreted as matter constituting an elementary particle.

condensor flux

conjunctor

conversion amplitude Allowing the transmutation of *photons into gravitophotons,* w_{ph_gp} (electromagnetic-gravitational interaction), and the conversion of *gravitophotons into quintessence particles,* w_{gp_q} (gravitational-gravitational interaction).

coupling constant Value for creation and destruction of messenger (virtual) particles, relative to the strong force (whose value is set to 1 in relation to the other coupling constants).

coupling potential between photon-gravitophoton (Kopplungspotential) As coupling potential the term $g_{ik}^{(gp)}$ of the metric is denoted. The reason for using the superscript *gp* is that this part of the photon metric equals the metric for the gravitophoton particle and that a →*sieve (conversion) operator* exists, which can transform a photon into a gravitophoton by making the second term in the metric anti-hermetric. In other words, the electromagnetic force can be transformed into a *repulsive* gravitational like force, and thus can be used to accelerate a material body.

cosmogony (Kosmogonie) The creation or origin of the world or universe, a theory of the origin of the universe (derived from the two Greek words kosmos (harmonious universe) and gonos (offspring)).

covariant For different inertial frames the laws of physics are varying so as to preserve the mathematical form of these laws. For instance, Newton's law of gravitation is *not* covariant under a →*Lorentz* transformation.

entelechy (Greek *entelécheia*, objective, completion) used by Aristotle in his work *The Physics.* Aristotle assumed that each phenomenon in nature contained an intrinsic objective, governing the actualization of a form-giving cause. The entelechial dimension can be interpreted as a measure of the quality of time varying organizational structures (inverse to entropy, e.g., plant growth) while the aeonic dimension is steering these structures toward a dynamically stable state. Any coordinates outside spacetime can be considered as steering coordinates.

differentiable manifold Contains a collection of points, each of which determines a unique position in →*Heim space*. The smoothness feature is only applicable in the case where the physical problem considered involves a large number of →*metrons*. Continuous and differentiable functions are supported. The differentiable manifold is a topological space (open sets) and is locally equivalent to a space $\mathbb{R}^n$ which is of the same dimension as the corresponding Heim space, i.e., there exists a one-to-one mapping between the open sets of the Heim space and the $\mathbb{R}^n$.

energy coordinates

epistemology Theory of the nature of knowledge especially with reference to its limits and validity.

eschatology Concerned with the final events in the history of the universe.

event

field activator (Feldkaktivator) flips the spin of a metron, i.e., changes its orientation.

flucton (Flukton) being movable and compressible.

flux aggregate

fundamental kernel (Fundamentalkern) Since the function $\kappa_{im}^{(\alpha)}$ occurs in $x_m^{(\alpha)} = \int \kappa_{im}^{(\alpha)} d\eta_i$ as the kernel in the integral, it is denoted as fundamental kernel of the *poly-metric*.

Galilean spacetime A spacetime in which the → *Galilei* transformation is valid

Galilei transformation

geodesic zero-line process This is a process where the square of the length element in a 6- or 8-dimensional →*Heim* space is zero.

gravitational limit(s) There are three distances at which the gravitational force is zero. First, at any distance smaller than *R*_, the gravitational force is 0. Second,

gravitophoton (field) Denotes a gravitational like field, represented by the metric sub-tensor, $g_{ik}^{(gp)}$, generated by a neutral mass with a smaller coupling constant than the one for gravitons, but allowing for the possibility that photons are transformed into gravitophotons. Gravitophoton particles can be both attractive and repulsive and are always generated in pairs from the vacuum under the presence of virtual electrons. The total enery extracted from the vacuum is zero, but only attractive gravitophotons are absorbed by protons or neutrons. The gravitophoton field represents the fifth fundamental interaction. The gravitophoton field generated by repulsive gravitophotons, together with the →*vacuum particle*, can be used to reduce the gravitational potential around a spacecraft.

graviton (Graviton) The virtual particle responsible for gravitational interaction.

heimline In analogy to →*worldline*, the path of a particle in →*Heim* space.

Heim-Lorentz force Resulting from the newly predicted gravitophoton particle that is a consequence of the →*Heim* space H^8. A metric subtensor is constructed in the subspace of coordinates I^2, S^2 and T^1, denoted as hermetry form H_5. The equation describing the Heim-Lorentz force has a form similar to the electromagnetic Lorentz force, except, that it exercises a force on a *moving body of mass m*, while the Lorentz force acts upon moving charged particles only. In other words, there seems to exist a direct coupling between matter and electromagnetism. In that respect, matter can be considered playing the role of charge in the Heim-Lorentz equation. The force is given by $\boldsymbol{F}_{gp} = \Lambda_p e \mu_0 \boldsymbol{v}^T \times \boldsymbol{H}$. Here Λ_p is a coefficient, $\boldsymbol{v}^T$ the velocity of a rotating body (insulator) of mass *m,* and **H** is the magnetic field strength. It should be noted that the gravitophoton force is 0, if velocity and magnetic field strength are parallel.

Heimian metric

Heim space A Heim space is a discrete space of 6, 8, or 12 dimensions, denoted as H^6, H^8, H^{12} respectively, with three spatial coordinates of + signature, while any other coordinate has - signature. In a Heim space an elemental surface, termed →*metron*, exists. If the surface considered comprises a large number of metrons, a Heim space can be considered a →*differentiable manifold* endowed with a →*Heimian metric*.

hermetry form (Hermetrieform) The word hermetry is an abbreviation of *hermeneutics*, in our case the semantic interpretation of the metric. To explain the concept of a hermetry form, the space H^6 is considered. There are 3 coordinate groups in this space, namely $s_3 = (\xi^1, \xi^2, \xi^3)$ forming the physical space $\mathbb{R}^3$, $s_2 = (\xi^4)$ for space T^1, and $s_1 = (\xi^5, \xi^6)$ for space S^2. The set of all possible coordinate groups is denoted by $S = \{s_1, s_2, s_3\}$. These 3 groups may be combined, but, as a general rule (stated here without proof, derived, however, by Heim from conservation laws in H^6, (see p. 193 in

Elementarstrukturen der Materie, Vol I, Resch Verlag 1996), coordinates ξ^5 and ξ^6 must always be curvilinear, and must be present in all metric combinations. An allowable combination of coordinate groups is termed *hermetry form*, responsible for a physical field or interaction particle, and denoted by *H*. *H* is sometimes annotated with an index, or sometimes written in the form $H=(\xi^1, \xi^2, ...)$. This is a symbolic notation only, and should not be confused with the notation of an n-tuple. From the above it is clear that only 4 hermetry forms are possible in H^6. It needs a →*Heim* space H^8 to incorporate all known physical interactions. Hermetry means that only those coordinates occurring in the hermetry form are curvilinear, all other coordinates remain Cartesian. In other words, *H* denotes the subspace in which physical events can take place, since these coordinates are non-euclidean. This concept is at the heart of Heim's geometrization of all physical interactions, and serves as the *correspondence principle* between geometry and physics.

hermeneutics (Hermeneutik) The study of the methodological principles of interpreting the metric tensor and the eigenvalue vector of the subspaces. This semantic interpretation of geometrical structure is called hermeneutics (from the Greek word to interpret).

hermitian matrix (self adjoint, selbstadjungiert) A square matrix having the property that each pair of elements in the *i*-th row and *j*-th column and in the *j*-th row and *i*-th column are conjugate complex numbers ($i \rightarrow -i$). Let A denote a square matrix and A^* denoting the complex conjugate matrix. $A^\dagger := (A^*)^T = A$ for a hermitian matrix. A hermitian matrix has real eigenvalues. If A is real, the hermitian requirement is replaced by a requirement of symmetry, i.e., the transposed matrix $A^T = A$.

homogeneous The universe is everywhere uniform and *isotropic* or, in other words, is of uniform structure or composition throughout.

hyperstructure (Hyperstruktur) Any lattice of a →*Heim* space that deviates from the isotropic Cartesian lattice, indicating an empty world, and thus allows for physical events to happen.

inertial transformation (Trägheitstransformation) Such a transformation, fundamentally an interaction between electromagnetism and the gravitational like gravitophoton field, reduces the inertial mass of a material object using electromagnetic radiation at specific frequencies. As a result of momentum and energy conservation in 4-dimensional spacetime, $v/c = v'/c'$, the Lorentz matrix remains unchanged. It follows that $c < c'$ and $v < v'$ where v and v' denote the velocities of the test body before and after the inertial transformation, and c and c' denote the speeds of light, respectively. In other words, since c is the vacuum speed of light, an inertial transformation allows for *superluminal speeds*. An inertial transformation is possible only in a 8-dimensional →*Heim* space, and is in accordance with the laws of SRT. In an Einsteinian universe that is 4-dimensional and contains only gravitation, this transformation does not exist.

isotropic The universe is the same in all directions, for instance, as velocity of light transmission is concerned measuring the same values along axes in all directions.

Lorentzian metric

Lorentz transformation This transformation in spacetime reflects the fundamental fact that light travels with exactly the same speed c with respect to any inertial frame

$$x' = \frac{x - vt}{(1 - v^2/c^2)^{1/2}}, \quad y' = y, z' = z,$$

$$t' = \frac{t - vx/c^2}{(1 - v^2/c^2)^{1/2}}.$$

Minkowski spacetime A spacetime in which the →*Lorentz* transformation is valid.

ontology A particular theory about the nature of being.

partial structure (Partialstruktur) For instance, in H^6, the metric tensor that is hermitian comprises three non-hermitian metrics from subspaces of H^6. These metrics from subspaces are termed partial structure.

poly-metric The term poly-metric is used with respect to the composite nature of the metric tensor in 8D →*Heim* space. In addition, there is the twofold mapping $\mathbb{R}^4 \rightarrow H^8 \rightarrow \mathbb{R}^4$.

probability amplitude With respect to the six fundamental interactions predicted from the →*poly-metric* of the →*Heim* space H^8, there exist six (running) coupling constants. In the particle picture, the first three describe gravitational interactions, namely w_g (graviton, attractive), w_{gp} (gravitophoton, attractive and repulsive), w_q (quintessence, repulsive). The other three describe the well known interactions, namely w_{ph} (photons), w_w (vector bosons, weak interaction), and w_s (gluons, strong interaction). In addition, there are two →*conversion amplitudes* predicted that allow the transmutation of *photons into gravitophotons* (electromagnetic-gravitational interaction), and the conversion of *gravitophotons into quintessence particles* (gravitational-gravitational interaction).

protosimplex

prototrope (Prototrope) first in time *protohistory* b : beginning : giving rise to *protoplanet

quantized bang According to Heim, the universe did not evolve from a hot big bang, but instead, spacetime was discretized from the very beginning, and such no infinitely small or infinitely dense space existed. Instead, when the size of a single *metron* covered the whole (spherical volume) universe, this was considered the beginning of this physical universe. That condition can be considered as the mathematical initial condition and, when inserted into Heim's equation for the evolution of the universe, does result in the initial diameter of the original universe [1]. Much later, when the *metron* size had decreased far enough, did matter come into existence as a purely geometrical phenomenon.

selector (Selektor)

shielding field (Schirmfeld)

sieve operator see → *transformtion operator*

transformation operator or sieve operator (Sieboperator) The direct translation of Heim's terminology would be *sieve-selector*. A transformation operator, however, converts a photon into a gravitophoton by making the coordinate ξ^4 Euclidean.

vacuum particle responsible for the acceleration of the universe, also termed quintessence particle The vacuum particle represents the *sixth* fundamental interaction, and is a *repulsive* gravitational force whose gravitational coupling constant is given by 4.3565×10^{-18} *G.* It only interacts with gravitons and positive (repulsive) gravitophotons, but not with real or virtual particles.

worldline Path of a particle in spacetime $\mathbb{R}^4$.

Appendix 3: Heim's Original Mathematical Terminology

Appendix 4: Mathematical Definitions

affine transformation is defined by a mapping (➔*lambda matrix*) from coordinates $x \rightarrow x'$: $x^{\mu'} = \Lambda^{\mu}_{\nu} x^{\nu} + a^{\mu}$ that guarantees the invariance of the spacetime interval $(\boldsymbol{x_B} - \boldsymbol{x_A})^2 - c^2 (t_B - t_A)^2 = 0$ between two ➔*worldpoints* A and B. It is necessary that the vacuum speed of light as an upper limit remains invariant between any two inertial systems. The ➔*Lorentz transformation* satisfies this requirement.

bijective mapping A mapping f is bijective if f *is both ➔injective and ➔surjective. The ➔inverse mapping* f^{1} is defined by $f\, x_0 \rightarrow x_0$ that is, f^{1} associates with each $y_0 \in Y$ the corresponding $x_0 \in D(f)$ for which $f\, x_0 = y_0$.

boost parameter

Casimir invariants the scalar product of ➔*Lie group generators* is known as *Casimir invariant* that commutes with all the generators (e.g., J^2 commutes with J_1, J_2, and J_3) and is therefore invariant under all group transformations. The eigenvalues of the *Casimir invariants* are the conserved quantum numbers of the symmetry group. The groups $O(3)$ and $SU(2)$ have only one invariant while the $SU(3)$ group has two invariants.

Clifford algebra in the Dirac equation four constant coefficients γ^{μ} occur that are non-commutable square matrices. Thus the wavefunction $\psi(x)$ is a *column matrix*. The γ matrices satisfy the condition $\{\gamma^{\mu}\gamma^{\nu}\} := \gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2\eta^{\mu\nu}$ where $\eta^{\mu\nu}$ is the Minkowski space-time metric tensor (diagonal form). The γ matrices are 4×4 matrices.

cotangent space consider a point P and its ➔*tangent space* $T_P M$ on a ➔*manifold* M. Let

$x^\mu(\lambda)$ be a smooth →*curve* through P on M. The directional derivative of a smooth function f on M at P (tangential vector **v**) with respect to the curve $x^\mu(\lambda)$ is a differentiable mapping $T_PM \rightarrow \mathbb{R}$. The set of all →*one-forms* $\omega(\boldsymbol{v})$ at P that map $T_PM \rightarrow \mathbb{R}$ forms the *dual vector space* to T_PM, termed the *cotangent space* T^*_PM. Bundling together all T^*_PM at different points on M, gives the *cotangent bundle* T^*M.

curve in a manifold M if λ is chosen to be the distance along the curve, a curve parametrized by λ has the tangent vector $(dx^\mu/d\lambda), \mu=1,2,\ldots,n.$

diffeomorphism A differentiable, bijective mapping f for which both f and f^{-1} are smooth, i.e., arbitrarily often differentiable (note: a homomorphism only requires that f and f^{-1} are continuous). It should be noted that a diffeomorphic mapping of generalized coordinates q to coordinates q' leaves the equations of motions invariant that are derived from Lagrange function L.

differentiable manifold A →*manifold* can be covered by patches (charts). Different coordinate systems can be set up on any part of a manifold. For two overlapping patches, two different coordinate systems can be defined in the overlap region, x^μ and $x^{\mu'} = x^{\mu'}(x^\mu)$. Any function f in the overlap region that is differentiable with respect to coordinates x^μ should also be differentiable with regard to $x^{\mu'}$. This is true if the transformation $x^{\mu'}(x^\mu)$ between the two coordinate systems is differentiable. Such a manifold is called a differentiable manifold. If there exist n derivatives there is a C^n manifold. For $n=\infty$, the manifold is C^∞.

dual space →*cotangent space.*

fiber bundle

groups:

compact Lie group a compact Lie group is a →*Lie group* whose parameters are defined in a closed interval, for instance, for **U(1)** angle θ varies in the interval $[0, 2\pi]$.

continuous groups contain an infinite number of elements. A simple example is the set of all complex wavefactors of a wavefunction in quantum mechanics written in the form $U(\theta)=e^{i\theta}$. The product of two phasefactors is $U(\theta)\,U(\theta')= U(\theta+\theta')$ and the inverse is given by $U^{-1}(\theta)=U(-\theta)$. These phase factors from a group called **U(1)**. This group is characterized by a single parameter, the angle θ in the interval $[0, 2\pi]$. The group is differentable since $dU = i\,U\,d\theta$ and thus the derivative is an element of the **U(1)**. The group of rotations in three-dimensional space the →***O(3)*** group and the group of →*Lorentz transformations* are also continuous groups.

Lie group the characteristic of a Lie group is that the parameters of a product are analytic functions of each parameter in the product that is, if $U(\theta) = U(\alpha)\,U(\beta)$ then $\theta=f(\alpha, \beta)$ where f is an analytic function. The analytic property guarantees that the group is differentiable so that an infinitesimal group element $dU(\theta)$ can be defined. The →*Lorentz group* is an example of a noncompact group. The boosts or transformations from one inertial frame to another are represented by non-unitary matrices. The →*boost parameter* $\eta=\tanh^{-1}(v/c)$ is not restricted to a closed interval.

Lorentz group

orthogonal groups O(n) is the group of rotations in an n-dimensional Euclidean space. The elements of $O(n)$ are $n\times n$ real valued matrices that have $n(n-1)/2$ independent parameters.

$O(3)$ is the three-dimensional rotation group and leaves the distance $x^2+y^2+z^2$ invariant. The parameters are the Euler angles α, β, γ

from classical mechanics. The rotation matrix is denoted by $R(\alpha, \beta, \gamma)$ and can be written as a sequence of three rotations.

Poincaré group

special unitary groups SU(n) has det $SU(n)=+1$ with $(n^2\text{-}1)$ independent elements.

special unitary group SU(2) and SU(3) represent two- and three-dimensional matrices and are associated with isotopic spin and color. A *SU(2)* rotation leaves the magnitude of the original vector invariant and has det *SU(2)*=1.

The *SU*(3) group has eight independent →*group generators*, represented by 3×3 →*hermitian matrices* and denoted as F_i, $i=1,...,8$. The matrices obey special commutation rules.

unitary group the elements of the unitary group $U(n)$ are given by $n\times n$ →*unitary matrices.* The determinant det $U(n)=\pm1$.

group generator:

Lie group generator let G be a Lie group and operators F_k be its group generator in analogy to the angular momentum operator J_k for the →*rotation group.* The operator F_k generates an element of G in the same way that J_k generates a rotation, i.e., $U=e^{-i\alpha_k F_k}$. The number of generators is equal to the number of independent parameters in G. Thus there are n(n-1)/2 generators for $O(n)$ and n^2-1 generators for $SU(n)$. Generators of the orthogonal and unitary groups are represented by →*hermitian* or self-adjoint matrices. The commutation rule is given by $[F_i, F_k]=i\, c_{ikm} F_m$. The c_{ikm} are called the structure constants. For $O(3)$ these values are either ±1 or 0. The structure of the is different from the group structure. The form the basis of a linear vector space that is known as *Lie algebra*. There exists both a scalar product and a vector product (in form of the commutation relation). This vector product is also called *Lie product.* For instance, for the total angular momentum $J^2=J_1^2+J_2^2+J_3^2$.

rotation group the rotation of wavefunction about a direction given by unit vector n is written as $R(\theta)=e^{-i\theta\hat{\boldsymbol{n}}\cdot\boldsymbol{J}}$, where $\boldsymbol{J}$ is the angular momentum operator. For an angle dθ, the rotation is given in first order as $R(\theta)=1-i\,d\theta\,\hat{\boldsymbol{n}}\cdot\boldsymbol{J}$. The combined rotations about the *x* and *y*-axis is given by $R(\theta)=(1-i\,d\theta\,J_1)(1-i\,d\phi\,J_2)$. Reversing the order of rotations and forming the difference is written as the commutator $[J_1, J_2]\, d\theta\, d\phi$. Angular momentum operators obey the commutation relation $[J_i, J_k]=i\,\varepsilon_{ikm} J_m$. Operators J_m are called the *group generators*. This means that $O(3)$ belongs to a non-commutative or non-Abelian group. In contrast, $O(2)$ and $U(1)$ are Abelian groups.

hermitian matrix

homeomorphism is a →*bijective mapping* $f: A\rightarrow B$ for which both f and f^{-1} are continuous.

homomorphism of O(3) and SU(2) there is a homomorphism (many-to-one mapping) between *O(3)* and *SU(2)* that is, a real three-dimensional rotation can be associated with an element of *SU(2)*. In other words, a vector (*x, y, z*) can be defined from the complex vector being transformed by *SU(2)* which is left invariant. It is possible to associate the general matrix $R(\alpha, \beta, \gamma)$ with the complex parameters of an *SU*(2) matrix. This means that the 3×3 real valued matrix O(3) can be expressed by the 2×2 complex valued matrix *SU*(2). However, the relation between the two matrices is not one-to-one, hence the denotation homomorphism (here two possibilities exist).

injective mapping a mapping is injective if for every $x_1 \neq x_2 \in \mathsf{D}(f)$, $x_1 \neq x_2$ implies $f\,x_1 \neq f\,x_2$ that is, different points in the domain have different images. Therefore, the inverse image is a single point in $\mathsf{D}(f)$.

inverse mapping for an →*injective mapping* $f: \mathsf{D}(f) \rightarrow Y$ the inverse mapping f^{1} is defined as to be the mapping $\mathsf{R}(f) \rightarrow \mathsf{D}(f)$ such that $y_0 \in \mathsf{R}(f)$ is mapped onto that $x_0 \in \mathsf{D}(f)$ for which $f\,x_0 = y_0$.

lambda matrix given are two coordinate systems x and x' on a manifold. The transformation matrix Λ, →*spinor,* is defined as

$$\Lambda^{\mu'}_{\mu} = \frac{\partial x^{\mu'}}{\partial x^{\mu}}.$$

Lie algebra →*group generator,* see *Lie group generator.*

Lorentz transformation this transformation (→*affine transformation*) in spacetime reflects the fundamental fact that light travels with exactly the same speed c with respect to any inertial frame (here x and x' denote coordinates)

$$x' = \frac{x - vt}{(1 - v^2/c^2)^{1/2}}, \quad y' = y, z' = z,$$

$$t' = \frac{t - vx/c^2}{(1 - v^2/c^2)^{1/2}}.$$

manifold A manifold is locally equivalent to an n-dimensional euclidean space $\mathbb{R}^n$. That is manifold M has the same local topology. M therefore must be a topological space, which means there is a collection of open sets that cover it. Second, the structure of these open sets, within small regions, is equivalent to the natural topology of $\mathbb{R}^n$. $\mathbb{R}^n$ is a Haussdorf space, which means that for any two different points there exist non-overlapping neighborhoods, and there exists a basis B in form of a collection of open sets such that each subset of $\mathbb{R}^n$ can be represented by a union of elements of B. Therefore for manifold M it is required that every point of M belongs to at least one open set of its basis B and there exists a one-to-one correspondence with the points of some open set of $\mathbb{R}^n$. This means that there is a continuous →*bijective mapping* from the open set of M to the open set in $\mathbb{R}^n$. When these conditions are met, M is called a manifold. Any function $f(P)$ for each point $P \in M$ can be re-expressed as a function $g(x^\mu)$ defined on $\mathbb{R}^n$. A manifold does not possess a metric, hence no scalar product can be defined on a manifold, →*one-form.*

mapping Given two sets X and Y with $A \subset X$. A mapping f from A into Y associates with each $x \in X$ a single $y \in Y$, called the *image* of x with respect to f, and written as $y = f\,x$. The set A is called the *domain,* $\mathsf{D}(f)$, of f. The *range* of f, $\mathsf{R}(f)$, is the set of all images. The inverse image of element $y_0 \in Y$ is the set of all $x \in \mathsf{D}(f)$ such that $f\,x = y_0$.

one-form one-forms are the extension of the scalar product of u v of two Euclidean vectors to a →*manifold M*. On M no metric is defined, therefore a salar product cannot be formed. A scalar product is considered as a function. For a given vector **u**, the symbol **u•** acts as a function that assigns to each vector **v** a real number. This mapping is linear. A *one-form* ω on M is therefore defined as a linear mapping that is real-valued. Because of the linearity $\omega(\mathbf{v}) = \omega_\mu v^\mu$. Indices of a one-form are in the lower position. A *one-form field* is defined in the same way as a linear function on vector fields. An example for a one-form field is the gradient of a scalar field f, denoted as $\partial_\mu f$. Taking as **v** the tangent vector d/dλ to a curve $x^\mu(\lambda)$, one can write

$$\omega_f(\mathbf{v}) = \frac{\partial f}{\partial x^\mu} \frac{\partial x^\mu}{\partial \lambda}.$$

partial derivative $\partial_\mu := \partial/\partial x^\mu$. The partial derivatives are sometimes also considered as *base vectors* in the corresponding coordinate systems, see →*one-form*.

rank of tensor field tensors and tensor fields are classified by their rank, denoted as $\binom{j}{i}$. Rank $\binom{0}{0}$ are called scalars and are real numbers. A scalar field is a real valued function *f*(*P*) on the manifold. Rank $\binom{1}{0}$ are called contravariant vectors. They correspond to a tangential vector on a curve in differential geometry. In a coordinate system a vector can be resolved in its components. In general the upper variable denotes the number of contravariant indices and the lower one the covariant indices of a tensor.

spinor the wavefunction of the Dirac function is a 4-component column matrix. These components will be expressed by a different set of functions and also be rearranged when transformed from coordinate system x to x. Since the components of are not components of a spacetime vector, but represent a state in which a particle can exist, they do not transform as vector components, i.e., they do not follow any tensor transformation law The transformation law for the ψ_β components is given by $\psi'_\alpha = S_{\alpha\beta}(\Lambda)\psi_\beta$. The matrix *S* is determined from the covariant form of the Dirac equation under a Lorentz transformation. For matrix Λ see →*lambda matrix.*

structure constants c_{ijk} →*Lie group generator.*

surjective mapping

tangent space consider a point *P* on a manifold *M*, for instance a point on a sphere embedded in $\mathbb{R}^3$. The set of all tangent vectors at *P* of all curves through *P* forms a vectorspace over $\mathbb{R}$, denoted by T_PM as the tangent space to *M* at *P*. All tangent vectors are in the tangential plane through *P*. For the example of the sphere T_PM is the vetorspace $\mathbb{R}^2$.

tangent bundle The disjoint union of all →*tangent spaces* $TM := \bigcup_P T_P M$ is itself a manifold of dimension *2n* if *M* has dimension *n*. The individual tangentspace T_PM at point *P* is called a *fiber*.

tensor field assigns a physical property to every point on the manifold.

unitary matrix (unitär) let *A* denote a square matrix, and *A** denoting the complex conjugate matrix. If $A^\dagger := (A^*)^T = A^{-1}$, then A is a unitary matrix, representing the generalization of the concept of orthogonal matrix. If *A* is real, the unitary requirement is replaced by a requirement of orthogonality, i.e., $A^{-1} = A^T$. The product of two unitary matrices is unitary.

unitary transformation used in quantum mechanics, leaving the modulus squared of the a complex wavefunction invariant, → *unitary group.*

world point, event, worldline a point (***x***, *t*) that specifies both the spatial coordinates and time is called a *world point* or *event*. The evolution of a signal, represented by a parametrized curve (***x***(*t*), *t*) is termed *worldline*.

[1] Woan, G., *The Cambridge Handbook of Physics Formulas*, Cambridge University Press, 2000.,